Programming Languages and Computation

Week 11: Undecidability and Reductions

* 1. [Trick question.] Is it decidable whether God exists?

* 2.

    1. Is the set $\emptyset$ decidable?

    2. Is the set $\mathbb{N}$ decidable?

** 3. Show that if $f : U \lesssim V$ and $g : V \lesssim W$ then $g \circ f : U \lesssim W$.

*** 4. Prove that the set
$$\text{ZERO} = \{\ulcorner S \urcorner \mid [\![S]\!]_x (0) \downarrow \}$$
is undecidable by reduction from HALT.

*** 5. Prove that the predicate ZERO is undecidable, but using Rice's theorem instead.

**** 6. Show that the predicate
$$U = \{\langle \ulcorner S \urcorner, \ulcorner T \urcorner \rangle \mid [\![S]\!]_x (0) \simeq [\![T]\!]_x (0)\}$$
is undecidable, by reduction from HALT.

*** 7. Is it possible to prove that the predicate U from the previous exercise is undecidable using Rice's theorem? If so, prove it. If not, explain why not.

**** 8. [Trick question.] Is the predicate
$$V = \{\ulcorner S \urcorner \mid \forall n \in \mathbb{N}.\ [\![S]\!]_x (n) \downarrow \}$$
from the last lecture semi-decidable? Why or why not?